Homework set 2: due Tuesday Oct 16th

1. A particle of mass $m$ is attached to a spring which produces a restoring force of magnitude $|F| = cx^3$, where $x$ is the distance from equilibrium and $c$ is a constant.
   (a) Set up the equation of motion of the particle and show a potential energy diagram illustrating the motion
   (b) Calculate the period of oscillations for a given amplitude $A$ and show that it is inversely proportional to $A$
   (c) If $c=10^4 \text{ Nm}^{-3}$ and $m=0.5 \text{ Kg}$, what is the period of oscillation for an amplitude of 2 cm.

2. A straight tunnel is bored through the earth connecting College Station to Beijing, China. If friction can be ignored, describe the type of motion that a capsule dropped in the tunnel will have. How long will it take to reach Beijing. If you make the tunnel connecting New York City instead of Beijing, how long would it take then? (Note: you will need to look up some data about the Earth, size, mass, etc.)

3. A simple pendulum consists of a spherical bob handing from a thread 1 m long. The upper end of the thread is held between your finger and thumb, and you move your had to and fro slightly in a horizontal plane at the natural frequency of the pendulum, to keep it swinging. You notice that the magnitude of the swing is 0.1 m when the amplitude of your hand motion is $2.5 \times 10^{-4}$ m. If the hand is now held still,
   (a) how long will it take for the amplitude to decay to $10/e$ cm ($e$ is the e-number)?
   (b) If you swing your hand at twice the natural frequency instead, how much would the amplitude of the bob be?

4. A compound pendulum consists of a thin cylindrical rod of length $l_1+l_2$ and uniform cross section; the pivot is at the end of $l_1$. The material above the pivot is a density $\rho_1$ and a coefficient of linear expansion $\alpha_1$, the corresponding quantities of the second material are $\rho_2$ and $\alpha_2$. Is it possible to make the period independent of temperature?

5. Recall the incline pendulum problem you worked out. Ignoring air resistance and assuming that only kinetic friction is the thing stopping the pendulum when you incline the plane at an angle $\alpha>0$. The board has a kinetic friction coefficient $\mu$ and the pendulum has a length $l$.
   a. Calculate the relation between the amplitudes between ticks. (hint: use the work energy theorem)?
   b. How many oscillations will you get for a given initial amplitude?
   c. What is the period of the oscillations?