PH221 Final Exam (team part)

Do 1 out of the 2 problems below.
Make your choice during the first five minutes; do not attempt both.

1) Recreating the Sun, melting lead, and blinding people:

A laser beam of diameter $d=2 \text{ mm}$ and power $P=1 \text{ mW}$ at a wavelength of $\lambda=630 \text{ nm}$ is focused by a lens of $f=5 \text{ mm}$ focal length onto the surface of an opaque material.

a) Neglecting thermal conduction, show that temperatures similar to that of the surface of the sun could be reached at the spot that the lens is illuminating.

b) Including thermal conductivity, estimate the temperature a block of lead would reach once the temperature reaches a steady state, i.e. the power input is balance by heat conduction to infinity. Will the lead be melted?

c) Now use the laser to point to someone’s eye. Estimate how fast you would reach a temperature of $300 \text{ C}^0$ at which point you will blind the person. You will need to estimate the focal length of the eye. Is it safe then to point to anyone’s eye with a laser? If so, for how long?

Stegran’s constant is $\sigma=5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$. The thermal conductivity of lead is $30 \text{ Wm}^{-1}\text{K}^{-1}$. The heat capacity of water is $4190 \text{ J Kg}^{-1}\text{ K}^{-1}$, the density of water is $1.0 \times 10^3 \text{ Kg m}^{-3}$

Hints: what determines the width of the focused beam if the source can be considered to be at infinity?
2) Rainbows in the glass planet:
You are now in another planet where the raindrops are made of ordinary flint glass (use graph below). Let’s take a look at the rainbows that are created. First the primary rainbow.

a) What is the relation between $\theta_i, \theta_r,$ and $\gamma_P$ in the figure below?

b) What is the maximum angle $\gamma_P$ for light with a wavelength of 550 nm? (you need to find the formula for the $\theta_i$ which will give you the maximum $\gamma_P$ and put it in the equation of part a).

c) Using the equations for the transmission and reflection coefficients below, what is the ratio of $t_{\perp}\text{final}/t_{||}\text{final}$ for the arch at this angle for light with a wavelength of 550 nm.

d) What is the angular width of the primary rainbow?
Do the same thing for the secondary rainbow:

e) What is the relation between $\theta_i, \theta_r$, and $\gamma_S$ in the figure below?

f) What is the maximum angle $\gamma_S$ for light with a wavelength of 550 nm? (you need to find the formula for the $\theta_i$ which will give you the maximum $\gamma_S$ and put it in the equation of part (e)).

g) Using the equations for the transmission and reflection coefficients below, what is the ratio of $t_{\perp,\text{final}}/t_{||,\text{final}}$ for the arch at this angle for light with a wavelength of 550 nm.

h) What is the angular width of the secondary rainbow?

Reflection and transmission coefficient for the different polarizations:

$$r_1 = \frac{E_{r,||}}{E_{0,||}} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_2 + n_2 \cos \theta_1}$$

$$r_\perp = \frac{E_{r,\perp}}{E_{0,\perp}} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

$$t_1 = \frac{E_{t,\parallel}}{E_{0,\parallel}} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1}$$

$$t_\perp = \frac{E_{t,\perp}}{E_{0,\perp}} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$