6.88 Solving (a) or (b) using the calculus of variation requires that you understand that the parametric variable that appears in the $J = \int J(g, y, \ldots; x) dx$ does not have to be a spatial variable as all the examples shown in the book. In fact, as you have seen, it is time $t$ that is used as an integration variable.

Then, what happens if we have $J(y, y'; \ldots, x)$ be independent of $x$? \( \Rightarrow \frac{d}{dt} (\frac{\partial J}{\partial y'}) = 0 \) and $\frac{\partial J}{\partial y} = 0$ is the thing that will maximize $J$. If there is a constrain function $g(y, y'; \ldots) = 0$, it appears the same as in 6.68 but now $\lambda$ does not depend on $x$ since $J$ does not ($J$ supposedly $g$ does not either).

Let's use this in our problem (let's do part b which is the more general case):

We want to maximize $V = 8xy^2$ with the restriction $g = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$. In terms of the calculus of variations is like saying that $x$ is parametrized by $t$.

We want to maximize $J = \int V(x(t), y(t), z(t); t) dt$ so we have

$$\frac{2V}{\partial x} - \frac{d}{dt} \frac{\partial V}{\partial x} + \lambda \frac{\partial g}{\partial x} = 0$$

and same for $y$ & $z$.

The set of equations to solve are then

\[
\begin{align*}
8y^2 + 2\lambda\frac{x}{a^2} = 0, \\
8x^2 + 2\lambda\frac{y}{b^2} = 0,
\end{align*}
\]

which can be combined in the eq's to get

$$x^2 = 16\lambda^2 \frac{a^4}{c^2} \Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} \Rightarrow \frac{V_{max}}{8a^2 b^2 c^2} = \frac{1}{16}$$

\[4 \text{ eq's} + 4 \text{ unknowns as needed} \]