Physics 302
Pre-test chapter 8

A spacecraft is in a circular orbit 200 km above Earth's surface. What is the minimum change in velocity Δv that can be applied as an impulse to completely escape from Earth orbit? In what direction should this impulse be applied, relative to the orbital velocity at that instant? Sketch the trajectory of such an escape, showing Earth and the satellite. What is the curve of that trajectory called? Write the equation of that orbit in suitable coordinates, and define the values of any coefficients. $g=9.8\ m/s^2$, $R_{Earth}=6400km$.

Solution: The minimum escape speed is obtained simply by requiring conservation of energy:

$$U_i + T_i = -\frac{GMm}{R_0} + \frac{1}{2} m v_e^2 = U_f + T_f = 0$$

$$v_e = \sqrt{\frac{2GM}{R_0}} \qquad g \equiv \frac{GM}{R_E^2}$$

$$v_e = R_E \sqrt{\frac{2g}{R_0}} = \left(6.4 \cdot 10^6 m\right) \sqrt{\frac{2 \cdot 9.8 m/s^2}{6.6 \cdot 10^6 m}} = 11 km/s$$

The velocity of the satellite in orbit is obtained by requiring orbit equilibrium:

$$\frac{m v_o^2}{R_0} = \frac{GMm}{R_0^2} \qquad v_0 = \sqrt{\frac{GM}{R_0}} = R_E \sqrt{\frac{g}{R_0}} = \frac{v_e}{\sqrt{2}}$$

The minimum Δv is obtained when the impulse is delivered parallel to the orbital velocity; in that case

$$\Delta v = v_e - v_o = \left(1 - \frac{1}{\sqrt{2}}\right) v_e = 3.2 km/s$$

The bounding trajectory between the family of ellipses (orbits) and hyperboli (open trajectories) is a *parabola*:

$$y = -R_0 + kx^2$$

Now we must find the appropriate value of k. We require that the acceleration at perigee ($x=0$ in the above $y(x)$) must be that produced by gravity:

$$\ddot{y} = 2k\left(\dot{x}^2 + x\ddot{x}\right) = 2k\dot{x}^2 = \frac{GM}{R_0^2}$$

$$\dot{x} = v_e \qquad k = \frac{GM}{2R_0^2 v_e^2} = \frac{1}{4R_0}$$

$$y = R_0\left(-1 + \frac{x^2}{4R_0}\right)$$